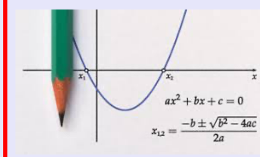
Math 125

Spring 2022

Lecture 24



Class QZ 17

Rationalize the denominator

1) $\frac{-2}{\sqrt{6}} = \frac{-2\sqrt{6}}{\sqrt{6}\sqrt{6}} = \frac{-2\sqrt{6}}{\sqrt{36}} = \frac{-2\sqrt{6}}{6} = \boxed{\frac{-\sqrt{6}}{3}}$ ✓

2) $\frac{3}{\sqrt{6}+\sqrt{3}} = \frac{3(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})} = \frac{3(\sqrt{6}-\sqrt{3})}{\sqrt{36}-\cancel{\sqrt{18}}+\cancel{\sqrt{18}}-\sqrt{9}} = \frac{\cancel{3}(\sqrt{6}-\sqrt{3})}{\underbrace{6-3}_{\cancel{3}}} = \boxed{\sqrt{6}-\sqrt{3}}$ ✓✓

1) Solve by Square-Root method:

$(2x-3)^2 = -98$

$2x - 3 = \pm\sqrt{-98}$

$2x - 3 = \pm\sqrt{49}\sqrt{2}\sqrt{-1}$

$2x = 3 \pm 7\sqrt{2}\,i$

$\boxed{x = \frac{3}{2} \pm \frac{7\sqrt{2}}{2} i}$

$\left\{\frac{3}{2} \pm \frac{7\sqrt{2}}{2} i\right\}$

2) Solve by Quadratic Formula:

$2x^2 + 5x - 18 = 0$

$ax^2 + bx + c = 0$ Quadratic Equation

$a=2$, $b=5$, $c=-18$

Discriminant $b^2 - 4ac = 5^2 - 4(2)(-18) = \boxed{169}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-5 \pm \sqrt{169}}{2(2)} = \frac{-5 \pm 13}{4}$

$x = \frac{-5+13}{4} = \frac{8}{4} = \boxed{2}$

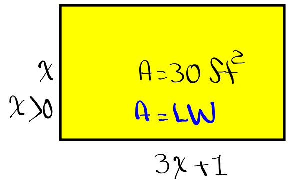
$x = \frac{-5-13}{4} = \frac{-18}{4} = \boxed{\frac{-9}{2}}$

$\left\{-\frac{9}{2}, 2\right\}$

Area of a rectangular garden is 30 ft^2.

The length is 1 ft longer than 3 times its width.

Find the dimensions of this garden.



$x(3x+1)=30$

$3x^2 + x - 30 = 0$

$a=3 \quad b=1 \quad c=-30$

$b^2-4ac = 1^2-4(3)(-30)$

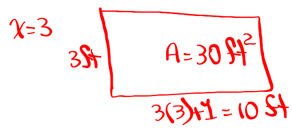
$= \boxed{361}$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{361}}{2(3)} = \frac{-1 \pm 19}{6}$$

$$x = \frac{-1+19}{6} = \frac{18}{6} = 3$$

$$x = \frac{-1-19}{6} = \frac{-20}{6} = \cancel{\frac{-10}{3}}$$

$x=3$



3 ft by 10 ft

Making a perfect square:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = \left(x + \frac{8}{2}\right)^2$$

$$x^2 + 8x + 4^2 = (x+4)^2$$

$$\boxed{x^2 + 8x + 16 = (x+4)^2}$$

$$x^2 - 6x + \left(\frac{6}{2}\right)^2 = \left(x - \frac{6}{2}\right)^2$$

$$x^2 - 6x + 3^2 = (x-3)^2$$

$$\boxed{x^2 - 6x + 9 = (x-3)^2}$$

Make a perfect-Square!

$$x^2 + 20x + \left(\frac{20}{2}\right)^2 = \left(x + \frac{20}{2}\right)^2$$

$$\boxed{x^2 + 20x + 100 = (x+10)^2}$$

$$x^2 - 12x + \left(\frac{12}{2}\right)^2 = \left(x - \frac{12}{2}\right)^2$$

$$\boxed{x^2 - 12x + 36 = (x-6)^2}$$

$$x^2 + 7x + \left(\frac{7}{2}\right)^2 = \left(x + \frac{7}{2}\right)^2$$

$$\boxed{x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2}$$

$$x^2 - 11x + \left(\frac{11}{2}\right)^2 = \left(x - \frac{11}{2}\right)^2$$

$$\boxed{x^2 - 11x + \frac{121}{4} = \left(x - \frac{11}{2}\right)^2}$$

$$x^2 + \frac{3}{5}x + \left(\frac{3}{10}\right)^2 = \left(x + \frac{3}{10}\right)^2$$

$$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$$

$$x^2 + \frac{3}{5}x + \frac{9}{100} = \left(x + \frac{3}{10}\right)^2$$

$$x^2 - \frac{4}{7}x + \left(\frac{2}{7}\right)^2 = \left(x - \frac{2}{7}\right)^2$$

$$\frac{1}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{7} = \frac{2}{7}$$

$$x^2 - \frac{4}{7}x + \frac{4}{49} = \left(x - \frac{2}{7}\right)^2$$

To make a pefect square:

Lead. Coef. must be 1

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Make a Perfect-Square:

$$3x^2 - 5x =$$

$$3\left(x^2 - \frac{5}{3}x + \left(\frac{5}{6}\right)^2\right) = 3\left(x - \frac{5}{6}\right)^2$$

$$\frac{1}{2} \cdot \frac{5}{3} = \frac{5}{6}$$

$$3\left(x^2 - \frac{5}{3}x + \frac{25}{36}\right) = 3\left(x - \frac{5}{6}\right)^2$$

Completing the Square method:

$$x^2 + 6x - 1 = 0$$

$$x^2 + 6x + 3^2 = 1 + 3^2$$

$$(x + 3)^2 = 10$$

Now use S.R.M.

$$x + 3 = \pm\sqrt{10}$$

$$\boxed{x = -3 \pm \sqrt{10}}$$

$$\{-3 \pm \sqrt{10}\}$$

Solve by completing the square method:

$$x^2 - 8x - 9 = 0$$

$$x^2 - 8x + 4^2 = 9 + 4^2$$

$$9 + 16$$

$$(x - 4)^2 = 25$$

Now S.R.M.

$$x - 4 = \pm\sqrt{25}$$

$$x = 4 \pm 5$$

$$x = 9, \quad x = -1$$

$$\{-1, 9\}$$

Solve by Completing the square method:

$x^2 + 10x + 29 = 0$

$x^2 + 10x + 5^2 = -29 + 5^2$

$\frac{10}{2} = 5$

$(x+5)^2 = -4$

Now by S.R.M.

$x + 5 = \pm\sqrt{-4}$

$\boxed{x = -5 \pm 2i}$

$\{-5 \pm 2i\}$

Solve by Completing the square method:

$2x^2 - 3x - 5 = 0$

$2x^2 - 3x = 5$

Divide by 2 to make Lead. Coef. 1

$\frac{2}{2}x^2 - \frac{3}{2}x = \frac{5}{2}$

$x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{5}{2} + \left(\frac{3}{4}\right)^2$

$\frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

$\left(x - \frac{3}{4}\right)^2 = \frac{5 \cdot 8}{2 \cdot 8} + \frac{9}{16}$

$\left(x - \frac{3}{4}\right)^2 = \frac{40}{16} + \frac{9}{16}$

$\left(x - \frac{3}{4}\right)^2 = \frac{49}{16}$

Use S.R.M.

$x - \frac{3}{4} = \pm\sqrt{\frac{49}{16}}$

$x = \frac{3}{4} \pm \frac{7}{4}$

$x = \frac{3+7}{4} = \frac{10}{4} = \boxed{\frac{5}{2}}$

$x = \frac{3-7}{4} = \frac{-4}{4} = \boxed{-1}$

$\left\{-1, \frac{5}{2}\right\}$

Solve by Completing the Square method:

$x^2 + 20x + 109 = 0$

$x^2 + 20x + 10^2 = -109 + 10^2$

$(x + 10)^2 = -9$

$x + 10 = \pm\sqrt{-9}$

$\boxed{x = -10 \pm 3i}$

$\{-10 \pm 3i\}$

How to determine the type of Solutions of a $\underbrace{\text{quadratic equation}}_{ax^2 + bx + c = 0,\ a \neq 0}$ without Solving:

1) Identify a, b, and c.

2) Compute the discriminant $b^2 - 4ac$

3) $b^2 - 4ac$
- > 0 Two real Solutions
- $= 0$ One repeated real Soln.
- < 0 Two complex-number Solns

Determine the type of Solutions for

$$4x^2 - 12x + 9 = 0$$

$a=4$ $b=-12$ $c=9$

$$b^2-4ac = (-12)^2 - 4(4)(9) = 0$$

Since $b^2-4ac=0$, we get one repeated real solution.

Determine the type of Solutions for

$$25x^2 + 20x + 6 = 0$$

$a=25$ $b=20$ $c=6$

$$b^2-4ac = 20^2 - 4(25)(6) = \boxed{-200}$$

Since $b^2-4ac<0$, we get two Complex-number Solutions.

Determine the type of Solutions for

$$(2x+5)(3x-1)=14.$$

Hint: FOIL, Simplify, write in $ax^2+bx+c=0$ form.

$$6x^2-2x+15x-5-14=0$$

$$6x^2+13x-19=0$$

$a=6$ $b=13$ $c=-19$

$$b^2-4ac=13^2-4(6)(-19)=\boxed{625}$$

Since $b^2-4ac>0$, we get two real Solutions.

How to find a quadratic equation when Solutions are given:

Find a quadratic equation in $ax^2+bx+c=0$ with solutions -4 and 5.

Solutions: $x=-4$ $x=5$

Factors: $x+4=0$ $x-5=0$ RHS $=0$

$$(x+4)(x-5)=0$$

$$x^2-5x+4x-20=0$$

$$\boxed{x^2-x-20=0}$$

Find a quadratic equation in $ax^2+bx+c=0$ form with solutions $-\frac{2}{3}$ and $\frac{1}{2}$.

Solutions $x=\frac{-2}{3}$ $x=\frac{1}{2}$

Clear fractions $3x=-2$ $2x=1$

Make RHS=0 $3x+2=0$ $2x-1=0$

Factors $(3x+2)(2x-1)=0$

Foil & Simplify $6x^2-3x+4x-2=0$

$$\boxed{6x^2+x-2=0}$$

Find a quadratic equation in $ax^2+bx+c=0$ form with solutions $3\pm\sqrt{5}$.

Solutions $x=3+\sqrt{5}$ $x=3-\sqrt{5}$

Make RHS=0 $x-3-\sqrt{5}=0$ $x-3+\sqrt{5}=0$

Factors $\underbrace{(x-3-\sqrt{5})(x-3+\sqrt{5})}_{\text{Conjugates}}=0$

$$(x-3)^2-(\sqrt{5})^2=0$$

$$(x-3)(x-3)-5=0 \quad \checkmark$$

Foil & Simplify $\rightarrow$ $\boxed{x^2-6x+4=0}$

Find a quadratic equation in $ax^2+bx+c=0$ form with solutions $-4 \pm 3i$.

Solutions $\quad x=-4+3i \quad\quad x=-4-3i$

Make RHS=0 $\quad x+4-3i=0 \quad\quad x+4+3i=0$

Factors $\quad \underbrace{(x+4-3i)(x+4+3i)}_{\text{Conjugates}}=0$

$$(x+4)^2-(3i)^2=0$$

$$\underbrace{(x+4)(x+4)}_{\text{Foil}}-9i^2=0$$

$$-9(-1)=0$$

$$\checkmark \quad \boxed{x^2+8x+25=0}$$

Find a quadratic equation in $ax^2+bx+c=0$ form with solutions $\frac{3}{5} \pm \frac{1}{5}i$.

Solutions $\quad x=\frac{3}{5}+\frac{1}{5}i \quad\quad x=\frac{3}{5}-\frac{1}{5}i$

Clear Fractions $\quad 5x=3+i \quad\quad 5x=3-i$

Make RHS=0 $\quad 5x-3-i=0 \quad\quad 5x-3+i=0$

Factors $\quad \underbrace{(5x-3-i)(5x-3+i)}_{\text{Conjugates}}=0$

$$(5x-3)^2-(i)^2=0$$

$$\underbrace{(5x-3)(5x-3)}_{\text{Foil}}-i^2=0$$

$$-(-1)$$

$$\boxed{25x^2-30x+10=0}$$

Class QZ 18

Solve by square-root method:

$$(2x-3)^2+10=-15$$

$$(2x-3)^2=-15-10$$

$$(2x-3)^2=-25$$

$$2x-3=\pm\sqrt{-25}$$

$$2x=3\pm 5i$$

$$\boxed{x=\frac{3}{2}\pm\frac{5}{2}i}$$

$$\left\{\frac{3}{2}\pm\frac{5}{2}i\right\}$$